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Keynes' Model as an Econometric Tool: Analysis of Macroeconomic Trends in the US from 1950s to 1980s

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Keynes' Model as an Econometric Tool: Analysis of Macroeconomic Trends in the US from 1950s to 1980s

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Resumo
O objetivo central deste texto é desenvolver o sistema analítico de Keynes em um modelo macro econométrico dinâmico, e este modelo é posteriormente aplicado à análise dos principais indicadores macroeconômicos dos EUA no período entre 1954 e 1984. No processo de desenvolver e aplicar o modelo macro, o sistema analítico de Keynes é revisto, assim como métodos econométricos usados para estimação de parâmetros de equações estruturais, e métodos matemáticos para a solução de sistemas de equações não lineares. Os resultados obtidos indicam que, embora os elementos da análise keynesiana sejam usados em geral de forma essencialmente teórica, este modelo econométrico baseado nesses elementos se mostrou capaz de reproduzir de forma adequada as principais tendências macroeconômicas nos EUA entre 1954 e 1984.

Palavras-chave: Modelos macro econométricos; Métodos para estimação e solução de sistemas de equações; Séries históricas e projeções macroeconômicas.

Abstract
The main goal of this paper is to develop the standard Keynes' analytical system into a dynamic macro-econometric model, which is then applied to the analysis of the main macroeconomic indicators for the US in the period from 1954 to 1984. In the process of developing and applying the macro model, the Keynesian analytical system is reviewed, as well as econometric methods used for parameter estimation of structural models, and mathematical methods for the solution of nonlinear systems of equations. The results obtained indicate that, although the elements of the Keynesian analysis are most often used in a purely theoretical perspective, this macro-econometric model based on those elements was successful in reproducing the main macroeconomic trends in the US from 1950s to 1980s.

Keywords: Macro-econometric modeling; Methods for estimation and solution of systems of equations; Macroeconomic forecasts; Financial investment decisions.

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Introduction

A clear observable fact is that mainstream macroeconomics – although the meaning of this expression is by no means certain – has evolved in the last few decades in the direction of bigger and more complex models devised to understand and analyze the behavior of the overall economy, and to make relevant conclusions about economic policy and future developments. As important examples of this statement, one can mention the predominance of large scale DSGE (Dynamic Stochastic General Equilibrium) models, as well as of VAR (Vector Auto-Regression) models. A problem that many economists and professionals perceive in these models is that they are not very successful in predicting future developments in macroeconomic aggregates, so that their practical use has been increasingly reduced.

Considering this predominant trend in Macroeconomics, this paper's main goal implies a move– quite radical, in fact –in the opposite direction. The objective is to use a structural macro-econometric model with barely a few simple equations, and evaluate its performance in reproducing major macroeconomic trends in a recent period in American economic history.

One of the pioneers in the field of structural macro-modeling was Lawrence R. Klein, who, in the 1940s, developed a six-equation dynamic macro-model, which became known as Klein Model I.[2]From the 1960s to 1980s, many well-known Econometrics textbooks contained thorough material on structural macro-dynamic models. One good example is Pindyck and Rubinfeld (1981). More recently, Greene (1997) includes a fairly detailed analysis of Klein Model I. A brief and interesting text that applies the developments in this field to macroeconomic planning is Heesterman (1970).

The model used here is developed from the standard Keynes' analytical system, and is then applied to the analysis of the main macroeconomic indicators for the US in the period from 1954 to 1984.[3] In the second Section, the Keynesian analytical system is reviewed and the econometric equations of the macro dynamic model are introduced.

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[2] The details on the data base, parameter estimation and solution of this model appear in Klein (1950). For more recent developments, one reference is Klein and Young (1982). In 1980, Klein was awarded the Nobel Prize for his contributions in the development and application of econometric models.
[3]It appeared previously in Fonseca (2005 and 2006).

In Section 3, econometric methods used for parameter estimation of structural models are briefly reviewed, and the estimates for the macro-econometric model are provided. Mathematical methods for the solution of nonlinear systems of equations are summed up in Section 4, and the nature of the solutions that were obtained is discussed in the following Section.

A simple model that transformed our understanding of market economies – and an econometric counterpart

From the involved and non-mathematical exposition in Keynes' General Theory, published in 1936, one can deduce a fairly simple and straightforward model with equations representing three overall markets: for goods, money and labor. The model's equations are specified below:[4]

I. Equilibrium output and aggregate production function

$$Y = C + I + G + (EX - IM) \quad (1)$$

$$Y = f(L) \quad (2)$$

II. Effective demand

$$C = C(Y - T) \quad (3)$$

$$I = I(r) \quad (4)$$

III. Nominal values

$$M = M(Y, r)P \quad (5)$$

$$w = \left[\frac{d}{dL} f \right] P \quad (6)$$

This system of equations is a staple tool in the Macroeconomics field and scarcely needs any further explanation. In any case, a description of the endogenous and exogenous variables follows:

[4] Although virtually any macroeconomic textbook contains the topics in this Section, the reference used here is Allen (1968), chap. 7.

Endogenous variables:

Y: Aggregate income and product, in real values;

L: Total employment (number of persons);

C: Private consumption, in real values;

I: Aggregate investment, in real values;

r: Interest rate, in real terms;

P: Average price level.

Exogenous variables:

G: Government spending, in real values;

$X–M$: Trade balance in national currency, real values;

T: Taxes, real values;

M: Money stock, current values;

w: Average wage rate, current values.

To a large extent, the contents of any standard Macroeconomics text book are related to the concepts and analyses arising from these six equations. In especial, the combination of equations (1), (3), (4), and (5) produces the standard IS-LM analysis of output and interest rate equilibrium. On the other hand, Equations (2) and (6) are usually dealt with independently of the rest of the system and, therefore, the price level and the labor force employed are treated in textbook analyses as exogenous elements.

In order to explore some of the economic rationality in this model, we can consider, for example, what would happen in a country where money stock (M) is allowed to increase. An initial effect would be a reduction of r, determined by eq.(5).[5] Then, given eq.(4), aggregate investment (I) would rise, leading to an increment in Y (eq. 1). This would cause an increase in C and further increments in Y through the multiplier effect. On the other hand, eq.(2) shows that a larger output depends upon a greater use of labor. However, from the standpoint of firms' profit maximization, a necessary condition for an increase in employment is that the cost of labor should be reduced, in order to compensate for the lower marginal productivity of labor (eq. 6, given that $f'' < 0$). This means that larger output and employment depend upon an increment of P, so that the real wage rate is reduced (workers behave as if they suffered

[5] The signs of the derivatives of these functions are not included, but this is straightforward information.

from monetary illusion, worrying only about the nominal wage rate w, at least in the short run). Thus the ultimate effects of an increase of M are increments in Y, C, I, L, and P, accompanied by a decrease of r.

Alternatively we can explore the consequences, derived from this six-equation system, of an increment in G. An initial effect, given eq. (1), would be a rise of Y. Moreover, if M is held fixed, eq.(5) indicates that r should increase, leading to a reduction of I (eq. 4). These opposing forces compensate each other, leaving Y relatively unchanged – the so-called *crowding out* effect of higher government spending. However, if M is allowed to increase in order to accommodate the demand pressures set off by an enlarged government spending, then the final results are very much like those derived in the previous paragraph.

In the sequence, a structural macro-econometric model based on Keynes' six-equation system is developed – a model that includes variables in both real and nominal values. To start, eq. (2) can be used in one of the other equations so that it can be eliminated – which makes sense given that the empirical development of an aggregate production function is an endeavor not commonly pursued. Introducing a relation for total employment, $L = f^{-1}(Y)$, and substituting in eq. (6), we get:

$$w = g(Y)\,P \tag{7}$$

Further, for the Keynesian system, one can introduce econometric equations that can be estimated using standard methods – that is, equations that are linear in the *parameters*. These econometric equations, however, are nonlinear in the endogenous *variables* – which means that one must use mathematical methods designed to solve nonlinear systems of equations.

IV. Econometric equations

$$Y = C + I + G + (EX - IM) \tag{1}$$

$$C = \alpha_0 + \alpha_1 (Y - T) + \varepsilon_C \tag{3a}$$

$$I = \beta_0 + \beta_1 r + \beta_2 \Delta Y + \beta_3 Y_{-1} + \varepsilon_I \tag{4a}$$

$$\frac{M}{P} = \gamma_0 + \gamma_1 r + \gamma_2 Y + \varepsilon_M \quad \rightarrow \quad r = \left(\frac{M}{P} - \gamma_0 - \gamma_2 Y - \varepsilon_M \right) \frac{1}{\gamma_1} \tag{5a}$$

$$\ln\left(\frac{w}{P}\right) = \zeta_0 + \zeta_1 \ln Y + \varepsilon_w \qquad (7a)$$

Eq. (4*a*) includes an endogenous variable with a one period delay and, consequently, this set of equations is a dynamic system – technically, a system of difference equations.[6] Therefore, this is a structural nonlinear dynamic macro-econometric model. But the fundamental question remains: Is it possible that such a simple system can be of value in analyses of the real world? Before we can try to answer, the equations must be estimated and solved. These developments are pursued in the next two sections.

Data set and estimation of the parameters – econometric methods for structural equations

Usually one of the most difficult and challenging tasks in the development of a macro-econometric model is to assemble the data base for its variables. However, in the case of the model developed in this paper, this was one of the easiest parts. The series used were drawn from data available on the covers of a pair of traditional textbooks – appropriately, one of the texts is on (Macro) Economics and the other on Econometrics.[7]These series appear in Appendix 1.

The estimation method used here is the Two-Stage Least Squares (2SLS) estimator, which is a particular case of the more general method known as Instrumental Variables (IV) estimator – which, in turn, can be considered an especial case of the Method of Moments applied to regression analysis.[8] It can be shown that estimators obtained fromthese methods have desirable asymptotical properties – further, under certain conditions, they are asymptotically efficient. The basic formula for the 2SLS estimator, applied to the *i*–th equation ($\bar{\square}_i$), is:

$$\bar{\square}_i = (\bar{Z}_i^T \bar{Z}_i)^{-1} \bar{Z}_i^T y_i \qquad (8)$$

[6] Eq. (4*a*), which shapes the dynamics of the model, plays a unique role in this system – it is actually an example of the "art" of specifying econometric equations. If the model's dynamics works appropriately, it should reproduce with some success the time path of the real macroeconomic variables.

[7] The references are: W. J. Baumoland A. S. Blinder, *Economics – Principles and Policy*, 3.ª ed., and Greene (1997).In the first reference's cover, there are data for *Y*, *C*, *I*, *G*, (*EX–IM*), nominal *r*, *w* and *P*. In the cover of the second, there are data for (*Y–T*) and*M*/*P*.

[8]Campbell, Lo and MacKinlay (1997).

In eq. (8), the data for the corresponding endogenous variable are included in y_i. Matrix $\bar{Z}_i$ is defined as:

$$\bar{Z}_i = [\hat{Y}_i | X_i] \qquad (9)$$

In this matrix, X_i contains the data for the exogenous (and lagged endogenous) variables included in the *i*–th equation, and the $\hat{Y}_i$ part contains the *instruments* – forecasts for the endogenous variables in the *i*–th equation, which were are generated using the model in its *reduced form*.[9] All the estimation procedures for the model's equations were developed in a spreadsheet file.

The parameters were estimated using data series for the 1953-79 period (Appendix 1) – the last five data available in each series were not used. The reason is that this part of the data was reserved so that it could be used to evaluate the model as a prediction tool – that is, beyond the period used in the estimation process. The estimates appear in Table 1.

Table 1 - Parameter estimates – 2SLS estimator
Data series: 1953-79.

Dependentvariable: *C*

Regressors	1	$Y–T$
Coefficient	8.052	0.8947
Standard error	5.297	0.0076

Dependent variable: *I*

Regressors	1	r	$\square Y$	Y_{-1}
Coefficient	–6.437	–3.3935	0.5975	0.1503
Standard error	12.591	4.2453	0.1182	0.0095

Dependent variable: *r*

Regressors	1	M/P	Y
Coefficient	1.1619	–0.0173	0.00423
Standard error	0.9955	0.00978	0.00277

Dependent variable: ln(*w/P*)

[9] Maddala (1987), chap. 11.

Regressors	1	ln$Y$
Coefficient	1.8317	0.3870
Standard error	0.1551	0.0226

Solving a system of nonlinear equations – each simulation period contains a problem to be solved

Solution methods of nonlinear systems of equations are based upon – with a decreasing level of generality – fixed-point theorems, the method of successive approximations, and the so-called Newton (or Newton-Raphson) method.[10]Fixed-point theorems establish that, under certain conditions, the solution of the equation $x = f(x)$ exists. One important aspect of this analysis is that any equation can be put in the form of the fixed-point equation. For example, if the original equation is $g(x) = 0$, then one corresponding $x = f(x)$ equation is:

$$x = x + g(x) \qquad (10)$$

The method of successive approximations is, at the same time, one way to prove the theorem and a method to find the solution. In this iterative scheme, one computes the successive approximations:[11]

$$x_{n+1} = f(x_n); \qquad n = 0, 1, 2, \dots \qquad (11)$$

The method associated to Newton is an important example of the iterations in (11). Representing the i–th equation in a nonlinear system by $g_i(\boldsymbol{x}) = 0$, where the symbol $\boldsymbol{x}$ represents the set (vector) of endogenous variables, the method provides the solution in each stage – or approximation – ($\boldsymbol{x}_{k+1}$) from the previous one ($\boldsymbol{x}_k$) through the following linear system:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - (J_k)^{-1} g_k \qquad (12)$$

[10]Franklin (1980), chap. 3,and Strang (1986), chap. 5.
[11] To prove the fixed-point theorem, it is sufficient to establish the convergence ofthe sequence x_n.

In eq. (12), J_k represents the Jacobian matrix (of first derivatives) obtained in stage k, and g_k is a vector with the results calculated at the same stage for the functions g_i. Usually, the Newton method converges quickly for the correct solution. One difficulty with this method is that the Jacobian matrix must be calculated at each step and, naturally, in a more convenient version of the method – albeit one in which convergence to the solution is not so quick, neither so much guaranteed – this matrix is fixed.In the so-called *modified* Newton method, only the Jacobian matrix for the first stage (0) is used:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - (J_0)^{-1} g_k \qquad (13)$$

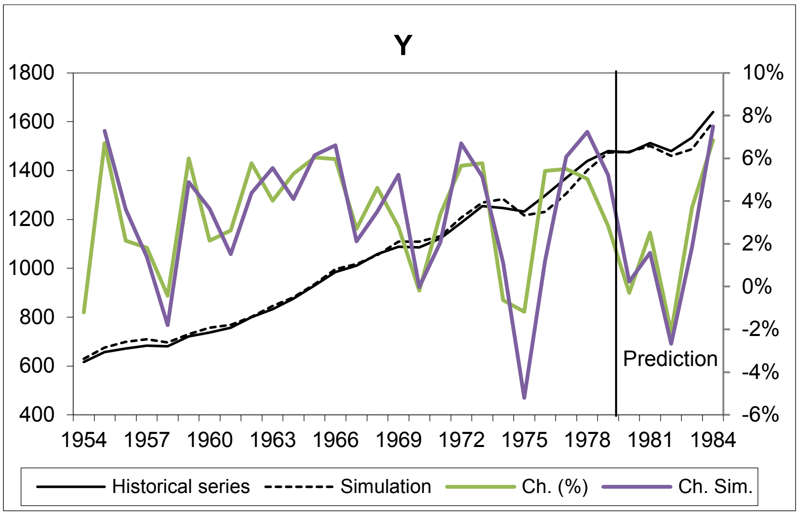
The modified Newton method is quite easilytranslated into computer code – in the case of the model solved here, a spreadsheet was used to provide the solutions. This arrangement is very convenient, since the construction of data series for the variables, the estimation of the equations' parameters, and the model's dynamic solution can all be performed in different spreadsheets of the same file.

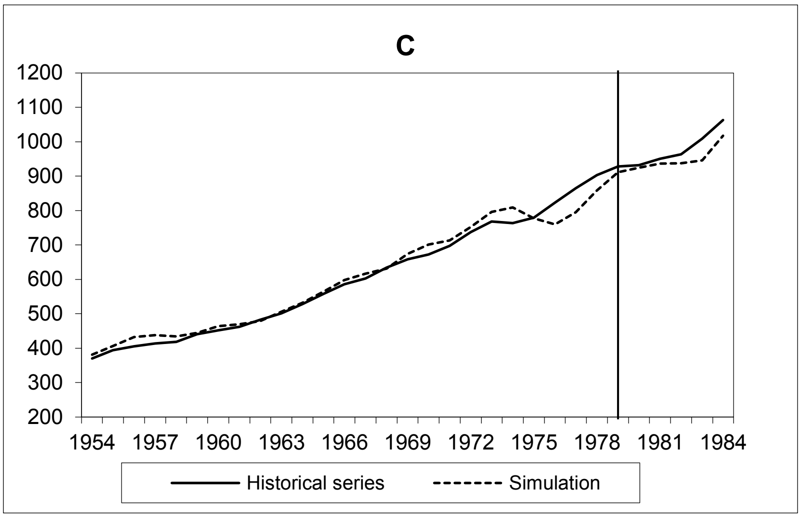
The first period for which the model was solved was 1954, usingdata for the previous year– which also provided a starting solution for the successive approximations method. After that, the solution obtained in one period was used as a starting point for the following year. In each period, the modified Newton method was utilized and, as a rule, only a few iterations were necessary to achieve convergence. Therefore, in terms of data for the endogenous variables, only information for 1953 was, in fact, used. In other words, the values for the endogenous variables obtained from the model reflect only the dynamic properties of the nonlinear system, and the trend of the exogenous variables – which are quite limited in number (only five).

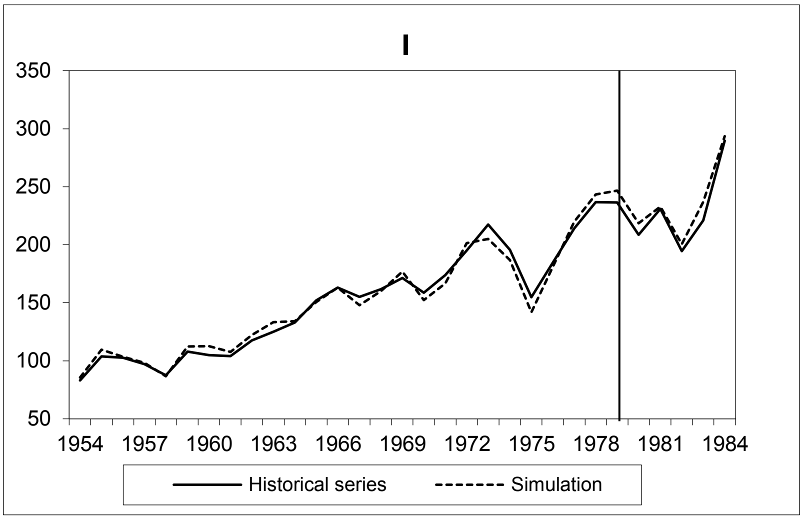
Analysis of the model's solutions: Macroeconomic trends in the American economy from the 1970s to 1980s

The solutions obtained from the five-equation macro-econometric model, together with the historical series, are displayed in Figure 1.

Figure 1 - Model's solutions and historical series

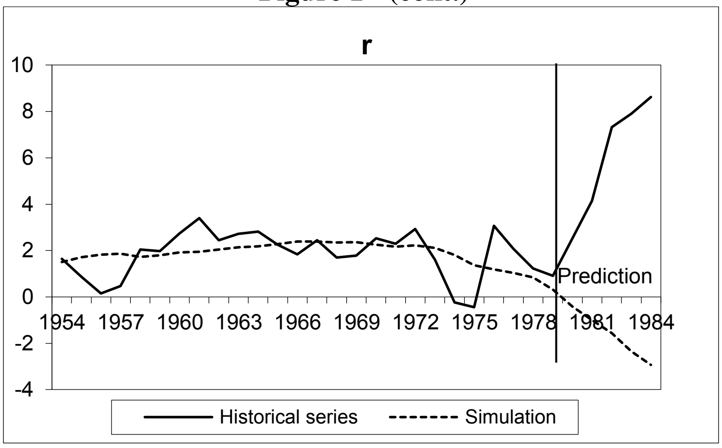


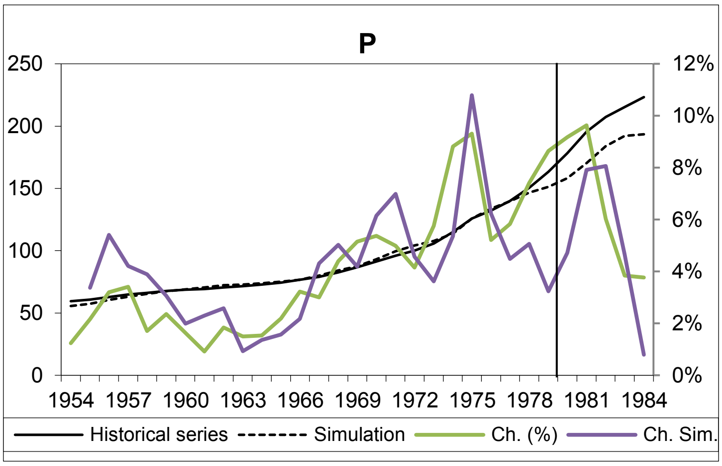


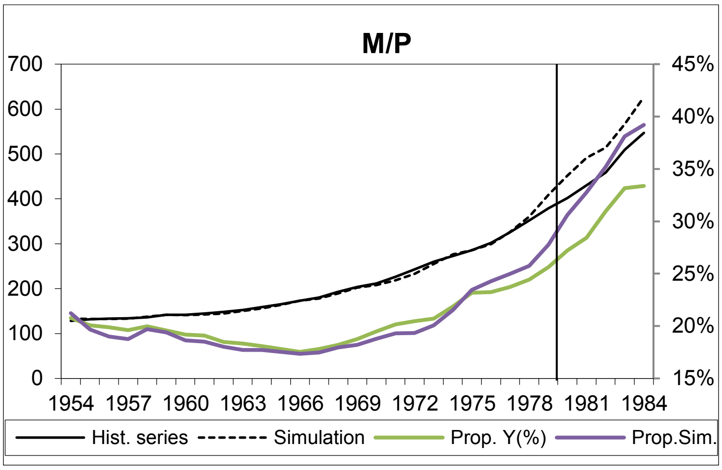


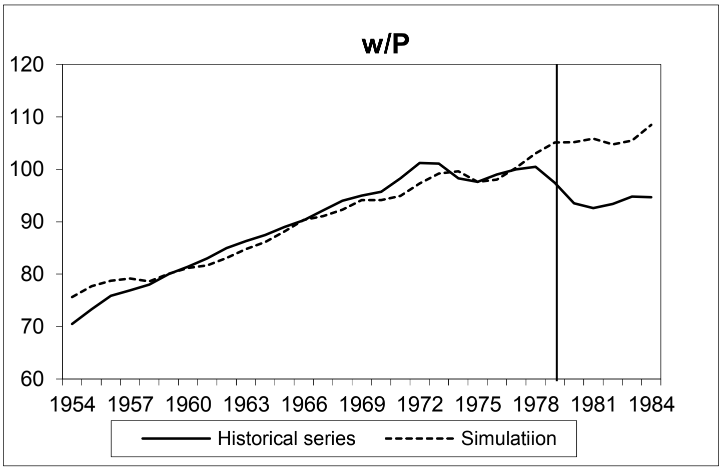
Notes: 1. For the units used in each graph, see Appendix 1.
2. The prediction period contains "out of sample" data – *i. e.*, data that were not used in parameter estimation.

Figure 1 - (cont.)









In Figure 1, the most significant graph is probably the one for aggregate investment (*I*) – the true dynamic variable in the model. Based on this graph, one can conclude that the model captured fairly well the dynamic pattern of the real economy.[12] On the other hand, one should not over emphasize this aspect of the model's solutions – that they were quite close to historical values –, given the especial nature of the exogenous variables. Although these variables are limited in number, each one represents fundamental and complex parts of the macroeconomic system.

For example, the solutions for the endogenous variables depend on the path of the nominal wage rate (*w*), which should be forecasted independently of the macro-model. Also, to predict the future path of trade balance (*EX–IM*), one would likely need a specific model oriented for the external sector. In any case, having said this, one favorable aspect of the macro-econometric model is that, if it were in fact used in

[12] Putting it differently, the econometric "artist" was successful.

analyses and predictions of macroeconomic series, the analyst could devise different scenarios, and the predictions could be revised as one of these scenarios revealed itself more likely to prevail in the future.

More over the most valuable feature of a structural macro-econometric model is that it reveals, among hundreds of different types of information available to researchers, what we should be concentrating on. More explicitly, if the model reproduces reasonably well the pattern of real macroeconomic variables, and given that the model's solutions derive from the exogenous variables and the model's dynamic properties, then it is a logical conclusion that these exogenous variables are the really important ones – and there are only five of them to keep track of.

In other words, one of the main accomplishments of this macro-econometric model is that it reveals that analysts and researchers should pay especial attention to the trends of average nominal wage, trade balance, government spending, taxes, and money supply. Further, the model produces *sophisticatedly simple* forecasts from the projected path of these variables – forecasts that can be of value in designing macroeconomic policies and strategies.

Concluding remarks

The analyses in the previous Sections indicate that structural macro-econometric models – even very simple ones, as the model examined here – can be valuable tools in the process of fore casting main macroeconomic variables and, in this way, they contain relevant information for macroeconomic analysts and researchers. On the other hand, it is important to emphasize that, as one would expect, there is no "magic bullet" here. Even though sophisticatedly simple macro-dynamic models can combine information in a systematic way, and generate solutions that would not normally be available otherwise, these solutions are derived from exogenous variables that depend on involved macroeconomic relations.

One drawback of structural macro-econometric models is that, beforethey can be applied to problems in the economic field and in Finance, a number of previous tasks must be successfully completed, in particular: development of relevant macro-econometric equations, building of data bases for the model's variables, estimation of the parameters using adequate econometric methods, and the application of mathematical methods for the solution of nonlinear systems of equations – in this case, there is also the need to write adequate computer code. In relation to these points, one favorable aspect is that sophisticated software tools are currently available to deal with all these tasks.

In the period that was examined here, the macro-econometric model based on Keynes was very successful in reproducing the endogenous variables, with the significant exception of the real exchange rate (*r*). The main conclusion is that there is convincing evidence that a simple structural dynamic macro-econometric model, as the one developed here, can be of help in designing macroeconomic policies and strategies.

Appendix

Data for major macroeconomic variables: USA, 1953-84[1]

	Y	*C*	*I*	*G*	*(EX-*	*(Y-T)*	r_{nom} [2]	*P* [3]	*M/P* [4]	*w* [5]
1953	623.6	363.4	85.3	170.1	4.8	399.1	1.62	58.82	126	39.93
1954	616.1	370	83.1	156	6.9	403.6	1.64	59.55	128	41.48
1955	657.5	394.1	103.8	152.3	7.3	427	0.87	60.84	132	44.07
1956	671.6	405.4	102.6	153.5	10.1	446.5	0.15	62.79	133.5	47.09
1957	683.8	413.8	97	161.2	11.8	455.2	0.47	64.93	134.1	49.34
1958	680.9	418	87.5	169.8	5.6	461	2.05	66.04	136	50.9
1959	721.7	440.4	108	170.6	2.7	479.3	1.97	67.6	141.4	53.44
1960	737.2	452	104.7	172.8	7.7	489.6	2.74	68.7	141.4	55.26
1961	756.6	461.4	103.9	182.9	8.5	503.9	3.4	69.33	144.5	56.86
1962	800.3	482	117.6	193.2	7.5	524.8	2.44	70.61	148	59.31
1963	832.5	500.5	125.1	197.6	9.4	542.7	2.72	71.67	152.6	61.12
1964	876.4	528	133	202.6	12.8	580.5	2.82	72.77	158.6	62.92
1965	929.3	557.5	151.9	209.8	10.1	616.3	2.26	74.36	165.5	65.4
1966	984.8	585.7	163	229.7	6.5	647	1.84	76.76	172.8	68.49
1967	1011.4	602.7	154.9	248.5	5.4	673.1	2.44	79.06	180	72.03
1968	1058.1	634.4	161.6	260.2	1.9	701.4	1.7	82.54	192.7	76.67
1969	1087.6	657.9	171.4	257.4	0.9	722.7	1.79	86.79	203.8	81.47
1970	1085.6	672.1	158.5	251.1	3.9	751.7	2.53	91.45	211.6	86.48
1971	1122.4	696.8	173.9	250.1	1.6	779.1	2.29	96.01	226.2	93.26
1972	1185.9	737.1	195	253.1	0.7	810.3	2.93	100	242.6	100
1973	1254.3	767.9	217.5	253.3	15.5	865.2	1.6	105.75	259.7	105.65
1974	1246.3	762.8	195.5	260.3	27.8	857.7	-0.23	115.08	272.6	111.78
1975	1231.6	779.4	154.8	265.2	32.2	874.8	-0.44	125.79	285.4	121.32
1976	1298.2	823.1	184.5	265.2	25.4	906.9	3.06	132.34	301.9	129.46
1977	1369.7	864.3	214.2	269.2	22	943.3	2.07	140.05	325.2	138.39
1978	1438.6	903.2	236.7	274.6	24	988.6	1.23	150.42	351.7	149.38
1979	1479.4	927.6	236.3	278.3	37.2	1015.5	0.91	163.42	379	157.28

1980	1475	931.8	208.5	284.3	50.3	1021.7	2.53	178.42	401.5	164.84
1981	1512.2	950.5	230.8	287	43.8	1049.7	4.14	195.6	430.1	178.98
1982	1480	963.3	194.4	292.7	29.7	1058.5	7.33	207.38	458.5	191.4
1983	1534.7	1009.2	221.1	291.9	12.6	1095.5	7.9	215.34	509.2	201.72
1984	1639.9	1062.4	289.6	302.1	-14.2	1169.1	8.62	223.44	547.3	209.09

1. Variables in billions of 1972 dollars, r_{nom} in %, *P*and*w*are indices (1972=100).
2. Average interest rates on bonds issued by top level companies.
3. GNP deflator.
4. M_1 stock.
5. Average hourly nominal wage.

Source: W. J. Baumol and A. S. Blinder, *Economics – Principles and Policy*, 3rd ed. (N. York, Harcourt Brace Jovanovich, 1985), and Greene(1997).

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